NAIC-ID(RS)T-0303-96

NATIONAL AIR INTELLIGENCE CENTER

ESTIMATION OF THE RANGE BETWEEN THE INTERCEPTER AND THE TARGET DURING INFRARED TERMINAL GUIDANCE OF SPACE INTERCEPTION

by

Shi Xiaoping, Wang Zicai, Ke Qihong

HUMAN TRANSLATION

NAIC-ID(RS)T-0303-96 8 July 1996

MICROFICHE NR:

ESTIMATION OF THE RANGE BETWEEN THE INTERCEPTER AND THE TARGET DURING INFRARED TERMINAL GUIDANCE OF SPACE INTERCEPTION

By: Shi Xiaoping, Wang Zicai, Ke Qihong

English pages: 11

Source: Hangkong Xuebao (Acta Aeronautica et Astronautica Sinica), Vol. 16, Nr. 3, May 1995; pp. 291-298

Country of origin: China
Translated by: Leo Kanner Associates
F33657-88-D-2188
Requester: NAIC/TASC/Richard A. Peden, Jr.
Approved for public release: distribution unlimited.

GRAPHICS DISCLAIMER

All figures, graphics, tables, equations, etc. merged into this translation were extracted from the best quality copy available.

Estimation of the Range between the Intercepter and the Target during Infrared Terminal Guidance of Space Interception

Shi Xiaoping, Wang Zicai, Ke Qihong

(Simulation Center, Harbin Institute of Technology, Harbin, 150001)

Abstract: A proper mathematical model is constructed according to the kinematic equations of space interception. After the component of the relative velocity normal to the line of sight is observed using a nonlinear observer design method with two canonical forms, the problem of estimating the range between interceptor and the target during infrared terminal guidance of space interception is solved. Numerical simulation verifies the above results.

Key Words: Space-interception, terminal guidance, optimal control

During infrared terminal guidance for space interception, the relative range between interceptor and target cannot be directly measured. Nonetheless, it has to be estimated since some guidance laws virtually rely on this physical quantity [1]. As far as nonlinear-observer design methods are concerned, most are subject to extremely rigorous application conditions. Here in this paper, a design method of this kind, based on two canonical forms, is introduced, which has proved to be applicable to the majority of nonlinear systems. Although the observer that our method produces may possibly contain a control variable derivative, a closed-loop observer without any control variable and its derivative can be created as long as the feedback control law is a continuous differentiable function of the state. The computing load of the foregoing design method depends on the

order and structure of the model. An appropriate model is presented in this paper, which serves in designing an observer to observe the relative velocity component in the direction normal to the line of sight, thus indirectly solving the problem of estimating the relative range between interceptor and target.

1. Constructing a Mathematical Model

The relative kinematic equation of interceptor and target is [1]

$$\left.\begin{aligned} \dot{r} &= v \\ \dot{v} &= r\omega^2 \\ \dot{\omega} &= -\frac{2v\omega}{r} - \frac{u_1}{r} \end{aligned}\right\} \tag{1}$$

where r is relative range; v is relative velocity; ω is visual angular velocity; u_1 is mobile acceleration of the interceptor. An infrared seeker head can only provide ω, whose value is generally at the 10^{-4}rad/s order of magnitude. In addition, v varies slightly, being basically a constant and known. From Eq. (1), the following can be derived:

$$\left.\begin{aligned} \frac{\mathrm{d}(r\omega)}{\mathrm{d}t} &= -v\omega - u_1 \\ \frac{\mathrm{d}(v\omega)}{\mathrm{d}t} &= r\omega^3 - \frac{v}{r}(2v\omega + u_1) \end{aligned}\right\} \tag{2}$$

and since $\omega^3 \approx 0$, state and control variables $x_1=r\omega$, $x_2=v\omega$, $u=v\omega$, $u=v\omega+u_1$, are introduced. As a result, a bivalent approximate model is obtained as follows:

$$\left.\begin{aligned} \dot{x}_1 &= -u = f_1(x_1,\ x_2,\ u) \\ \dot{x}_2 &= -\frac{x_2}{x_1}(x_2 + u) = f_2(x_1,\ x_2,\ u) \\ y &= x_2 = h(x_1,\ x_2,\ u) \end{aligned}\right\} \tag{3}$$

where the measurement equation selected is rational. According to Eq. (3), the observer is now designed to observe the relative velocity component in the direction normal to the line of sight x_1, and then to estimate r. The visibility matrix [2] of this system is

$$Q(x_1,\ x_2,\ u)=\begin{bmatrix} 0 & 1 \\ \dfrac{x_2^2+x_2u}{x_1^2} & -\dfrac{2x_2+u}{x_1} \end{bmatrix} \tag{4}$$

Thereby, the systematic equation (3) is visible as long as

$x_1x_2(x_2+u)\neq 0$.

2. Design of Nonlinear State Observer

For the convenience of design, first suppose the canonical form I of the systematic equation (3)

$$\left.\begin{aligned} \dot{x}_1^* &= x_2^* = f_1^*(x_1^*,\ x_2^*,\ u,\ \dot{u},\ \ddot{u}) \\ \dot{x}_2^* &= -a^*(x_1^*,\ x_2^*,\ u,\ \dot{u},\ \ddot{u}) = f_2^*(x_1^*,\ x_2^*,\ u,\ \dot{u},\ \ddot{u}) \\ y &= h^*(x_1^*,\ x_2^*) = x_1^* \end{aligned}\right\} \tag{5}$$

The conversion from Eq. (3) to canonical form I is[2]

$$\left.\begin{aligned} q_1(x_1,\ x_2,\ u,\ \dot{u}) &= h(x_1,\ x_2,\ u) = x_1^* \\ q_2(x_1,\ x_2,\ u,\ \dot{u}) &= \hat{N}q_1(x_1,\ x_2,\ u,\ \dot{u}) = x_2^* \end{aligned}\right\} \tag{6}$$

where the operator $\hat{N}$ is

$$\hat{N}h=\left[\frac{\partial h}{\partial u}\ \frac{\partial h}{\partial \dot{u}}\right]\begin{bmatrix}\dot{u}\\ \ddot{u}\end{bmatrix}+\left[\frac{\partial h}{\partial x_1}\ \frac{\partial h}{\partial x_2}\right]f \tag{7}$$

where $f=[f_1,\ f_2]^T$; for the given problem in this paper, the conversion equation (6) is

$$\left.\begin{aligned} x_1^{*} &= x_2 \\ x_2^{*} &= -\frac{x_2^2 + x_2 u}{x_1} \end{aligned}\right\} \tag{8}$$

Therefore, the characteristic nonlinear function in Eq. (5) is

$$a^{*}(x_1^{*},\ x_2^{*},\ u,\ \dot{u},\ \ddot{u}) = -\frac{2}{x_1^{*} + u} x_2^{*2} - \frac{\dot{u}}{x_1^{*} + u} x_2^{*} \tag{9}$$

Since the observer cannot be directly constructed from Eq. (5), it should be converted to the canonical form II

$$\left.\begin{aligned} \dot{x}_1^{\Delta} &= -a_0(y,\ u,\ \dot{u},\ \ddot{u}) \\ \dot{x}_2^{\Delta} &= x_1^{\Delta} - a_1(y,\ u,\ \dot{u}) \\ x_2^{\Delta} &= c(y,\ u) \end{aligned}\right\} \tag{10}$$

Assume that the conversion from Eqs. (3) to (10) is

$$\left.\begin{aligned} x_1^{\Delta} &= v_1(x_1,\ x_2,\ u,\ \dot{u}) \\ x_2^{\Delta} &= v_2(x_1,\ x_2,\ u) \end{aligned}\right\} \tag{11}$$

where $[x_1,\ x_2]^T \in G$, and v_1, v_2 are continuous differentiable functions of x_1, x_2, u and $\dot{u}$; the total derivative with regards to t in the foregoing equation should be solved and Eq. (10) should be taken into consideration. After all these procedures, the following can be finally arrived at:

$$\left.\begin{aligned} O &= Mc^2(y,\ u) + Ma_1(y,\ u,\ \dot{u}) + a_0(y,\ u,\ \dot{u},\ \ddot{u}) \\ v_1(x_1,\ x_2,\ u,\ \dot{u}) &= Mc(y,\ u) + a_1(y,\ u,\ \dot{u}) \\ v_2(x_1,\ x_2,\ u) &= x_2^{\Delta} = c(y,\ u) \end{aligned}\right\} \tag{12}$$

where operator M is

$$M\zeta = \begin{bmatrix} \frac{\partial \zeta}{\partial x_1} & \frac{\partial \zeta}{\partial x_2} \end{bmatrix} f + \begin{bmatrix} \frac{\partial \zeta}{\partial u} & \frac{\partial \zeta}{\partial \dot{u}} \end{bmatrix} \begin{bmatrix} \dot{u} \\ \ddot{u} \end{bmatrix} \tag{13}$$

where ζ is a scalar continuous differentiable function. Combining Eqs. (12) with (5) and (9), the following is derived:

$$O = M^{*2} c(y, u) + M^{*} a_1(y, u, \dot{u}) + a_0(y, u, \dot{u}, \ddot{u}) \tag{14}$$

where operator M^* is

$$M^{*}\zeta = \left[\frac{\partial \zeta}{\partial x_1^{*}} \; \frac{\partial \zeta}{\partial x_2^{*}}\right] f^{*} + \left[\frac{\partial \zeta}{\partial u} \; \frac{\partial \zeta}{\partial \dot{u}}\right] \begin{bmatrix} \dot{u} \\ \ddot{u} \end{bmatrix} \tag{15}$$

where $f^{*} = [f_1^{*} \; f_2^{*}]^{T}$; by substituting Eq. (5) in the foregoing equation and then in Eq. (14), the following is acquired:

$$\begin{aligned} O = {} & \frac{\partial^2 c(y, u)}{\partial y^2} x_2^{*2} - \frac{\partial c(y, u)}{\partial y} a^{*}(x_1^{*}, x_2^{*}, u, \dot{u}, \ddot{u}) + \frac{2\partial^2 c(y, u)}{\partial y \partial u} \dot{u} x_2^{*} + \\ & \frac{\partial^2 c(y, u)}{\partial u^2} (\dot{u})^2 + \frac{\partial c(y, u)}{\partial u} \ddot{u} + \frac{\partial a_1(y, u, \dot{u})}{\partial y} x_2^{*} + \\ & \frac{\partial a_1(y, u, \dot{u})}{\partial u} \dot{u} + \frac{\partial a_1(y, u, \dot{u})}{\partial \dot{u}} \ddot{u} + a_0(y, u, \dot{u}, \ddot{u}) \end{aligned} \tag{16}$$

By substituting Eq. (9) in the foregoing equation, the following partial differential coupled equations can be obtained as follows:

$$\left.\begin{aligned} & -\frac{2}{y+u} \frac{\partial c(y, u)}{\partial y} = \frac{\partial^2 c(y, u)}{\partial y^2} \\ & -\frac{\dot{u}}{y+u} \frac{\partial c(y, u)}{\partial y} = \frac{2\partial^2 c(y, u)}{\partial y \partial u} \dot{u} + \frac{\partial a_1(y, u, \dot{u})}{\partial u} \\ & -a_0(y, u, \dot{u}, \ddot{u}) = \frac{\partial c(y, u)}{\partial u} \ddot{u} + \frac{\partial a_1(y, u, \dot{u})}{\partial \dot{u}} \ddot{u} + \\ & \qquad \frac{\partial a_1(y, u, \dot{u})}{\partial u} \dot{u} + \frac{\partial^2 c(y, u)}{\partial u^2} (\dot{u})^2 \end{aligned}\right\}$$

From the foregoing coupled equations, a set of simplest solutions can be derived:

$$\left.\begin{aligned} & c(y, u) = -\frac{2}{y+u} \\ & a_1(y, u, \dot{u}) = -\frac{3\dot{u}}{(y+u)^2} \\ & a_0(y, u, \dot{u}, \ddot{u}) = \frac{\ddot{u}}{(y+u)^2} - \frac{2}{(y+u)^3} (\dot{u})^2 \end{aligned}\right\} \tag{17}$$

Substituting the foregoing equation in Eq. (12), the conversion equation (11) is acquired

$$\left.\begin{aligned} x_1^{\Delta} &= -\frac{2x_2}{x_1(x_2+u)} - \frac{\dot{u}}{(x_2+u)^2} \\ x_2^{\Delta} &= -\frac{2}{x_2+u} \end{aligned}\right\} \tag{18}$$

By substituting Eq. (17) in Eq. (10), the specific canonical form II is obtained as follows:

$$\left.\begin{aligned} \dot{x}_1^{\Delta} &= -\frac{\ddot{u}}{(y+u)^2} + \frac{2}{(y+u)^3}(\dot{u})^2 \\ \dot{x}_2^{\Delta} &= x_1^{\Delta} + \frac{3\dot{u}}{(y+u)^2} \\ x_2^{\Delta} &= -\frac{2}{y+u} \end{aligned}\right\} \tag{19}$$

The observer of the systematic equation (19) is

$$\left.\begin{aligned} \dot{\hat{x}}_1^{\Delta} &= -\frac{\ddot{u}}{(y+u)^2} + \frac{2}{(y+u)^3}(\dot{u})^2 - p_1\left(\frac{2}{y+u} + \hat{x}_2^{\Delta}\right) \\ \dot{\hat{x}}_2^{\Delta} &= \hat{x}_1^{\Delta} + \frac{3\dot{u}}{(y+u)^2} - p_2\left(\frac{2}{y+u} + \hat{x}_2^{\Delta}\right) \end{aligned}\right\} \tag{20}$$

By subtracting Eq. (20) from Eq. (19), the state error equation is obtained:

$$\left.\begin{aligned} \dot{\tilde{x}}_1^{\Delta} &= -p_1\tilde{x}_2^{\Delta} \\ \dot{\tilde{x}}_2^{\Delta} &= \tilde{x}_1^{\Delta} - p_2\tilde{x}_2^{\Delta} \end{aligned}\right\} \tag{21}$$

where $\tilde{x}_i^{\Delta} = x_i^{\Delta} - \hat{x}_i^{\Delta} (i = 1,\ 2)$. The parameters selected are $p_1=\lambda_1\lambda_2$, $p_2=-(\lambda_1+\lambda_2)$, where λ_1 and λ_2 are poles of the systematic-error

equation (21). Under the action of the conversion equation (18), the observer of systematic equation (3) can be acquired from Eq. (20)

$$\left.\begin{aligned}\dot{\hat{x}}_1 &= \frac{(\hat{x}_2+u)\hat{x}_1^2}{2\hat{x}_2}\left[\frac{2(\dot{u})^2}{(y+u)^3} - \frac{\ddot{u}}{(y+u)^2} + 2p_1\left(\frac{1}{\hat{x}_2+u} - \frac{1}{y+u}\right)\right] + \\ &\quad \frac{(u^2+u\hat{x}_2-\dot{u}\hat{x}_1)\hat{x}_1}{2\hat{x}_2}\left[\frac{3\dot{u}}{(y+u)^2} - \frac{2\hat{x}_2}{\hat{x}_1(\hat{x}_2+u)} - \frac{\dot{u}}{(\hat{x}_2+u)^2} + \right. \\ &\quad \left. 2p_2\left(\frac{1}{\hat{x}_2+u} - \frac{1}{y+u}\right)\right] + \frac{(\ddot{u}\hat{x}_1 - 2\dot{u}\hat{x}_2 - 2u\dot{u})\hat{x}_1}{2\hat{x}_2(\hat{x}_2+u)} \\ \dot{\hat{x}}_2 &= \frac{(\hat{x}_2+u)^2}{2}\left[\frac{3\dot{u}}{(y+u)^2} - \frac{2\hat{x}_2}{\hat{x}_1(\hat{x}_2+u)} + 2p_2\left(\frac{1}{\hat{x}_2+u} - \frac{1}{y+u}\right)\right] - \frac{3}{2}\dot{u}\end{aligned}\right\} \tag{22}$$

3. Estimation of Relative Range between Interceptor and Target

To eliminate $\dot{u}$, $\ddot{u}$, and u in the foregoing equation, the visual equation (3) is a monovalent system. In accordance with the principle of designing a guidance law, r and ω should be made to approach zero so as to give the index

$$J = \frac{1}{2}r_1x_1^2(t_f) + \frac{1}{2}\int_{t_0}^{t_f}[q_1x_1^2(t) + s_1u^2(t)]\mathrm{d}t \tag{23}$$

then the optimal control can be solved as

$$u(t) = \frac{1}{s_1}p(t)x_1(t) \tag{24}$$

where p(t) can satisfy Riccati differential equation

$$\dot{p}(t) = \frac{1}{s_1}p^2(t) - q_1 \tag{25}$$

the final value condition is $p(t_f)=r_1$. The solution to that equation is

$$p(t)=\sqrt{s_1 q_1}\;\frac{(r_1+\sqrt{s_1 q_1})+(r_1-\sqrt{s_1 q_1})\exp\left[2\sqrt{\frac{q_1}{s_1}}(t-t_f)\right]}{(r_1+\sqrt{s_1 q_1})-(r_1-\sqrt{s_1 q_1})\exp\left[2\sqrt{\frac{q_1}{s_1}}(t-t_f)\right]} \tag{26}$$

Based on Eqs. (3), (6) and (25), the following can be derived:

$$\dot{u}(t)=-\frac{q_1}{s_1}x_1(t) \tag{27}$$

$$\ddot{u}(t)=\frac{q_1}{s_1^2}p(t)x_1(t) \tag{28}$$

From Eqs. (24), (27) and (28), select

$$\left.\begin{aligned} u&=\frac{1}{s_1}p(t)\hat{x}_1 \\ \dot{u}&=-\frac{q_1}{s_1}\hat{x}_1 \\ \ddot{u}&=\frac{q_1}{s_1^2}p(t)\hat{x}_1 \end{aligned}\right\} \tag{29}$$

By substituting the foregoing equation in Eq. (22), the closed-loop observer can be constructed. The visual relative velocity v is a constant, then from

$$\hat{x}_1=\hat{r}\omega \tag{30}$$

the relative range can be estimated as

$$\hat{r}=\frac{\hat{x}_1}{\omega}=\frac{v\hat{x}_1}{v\omega}=\frac{v\hat{x}_1}{y}. \tag{31}$$

Fig. 1 shows the structure of state estimation and the generation

of relative range estimation.

4. Numerical Simulation Check

During a numerical simulation, select $r_1=0.4$, $q_1=0.3$, $s_1=0.3$, $\lambda_1=-20$, $\lambda_2=-30$. Selecting $t_0=0$, t_f is the blind region entry moment, $r(0)=150$km, $v=-7.5$km/s, $\omega(0)=5\times10^{-4}$rad/s, then $x_1(0)=75$m/s, $x_2(0)=-3.75$m/s^2; select $\hat{x}_1(0)=80$m/s, $\hat{x}_2(0)=-4$m/s^2. Figures 2-5 show the desirable numerical simulation results.

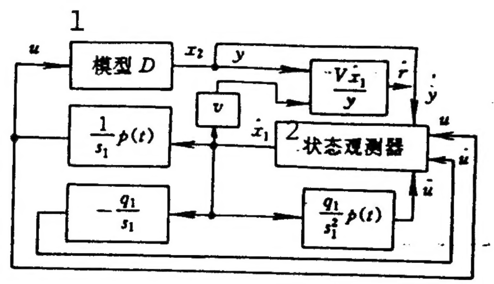

Fig. 1. Structure of State Estimation

Key: 1. Model D; 2. State observer

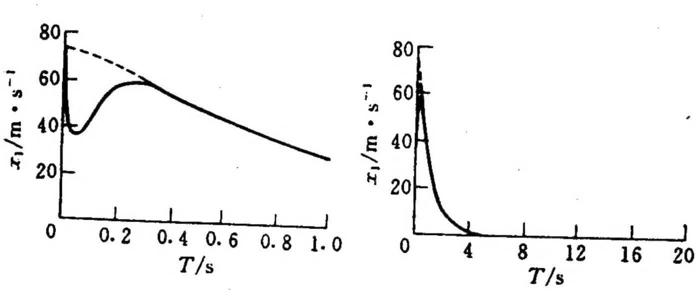

Fig. 2 State x_1 and Its Estimated Value $\hat{x}_1$

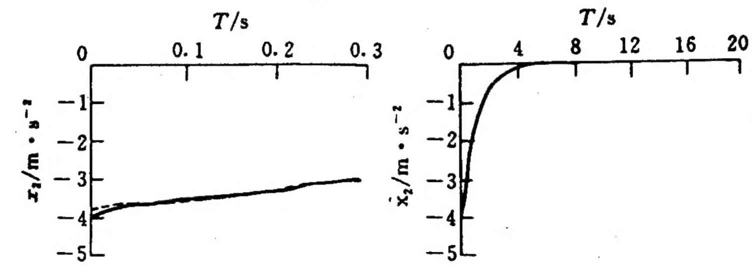

Fig. 3 State x_2 and Its Estimated Value $\hat{x}_2$

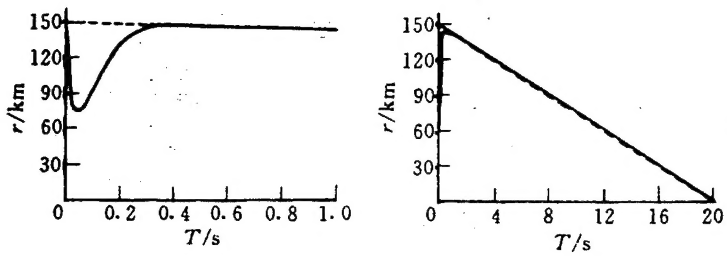

Fig. 4 Relative Range r and Its Estimated Value $\hat{r}$

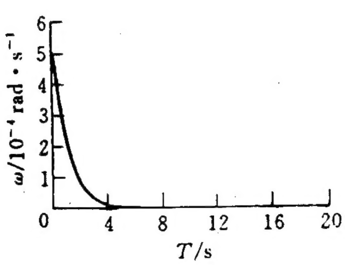

Fig. 5 Visual Angular Velocity ω

5. Conclusions

By using our nonlinear state observer design method in two canonical forms, estimation of the relative range between interceptor and target during infrared terminal interception was indirectly made through an appropriate model and confirmed by

numerical simulation results. As for the stability of the closed-loop nonlinear feedback system with an observer, see reference [3].

REFERENCES

1 史小平, 王子才. 空间拦截最优末端导引规律研究. 宇航学报, 1993; (3): 18–25

2 Xia X H, Gao W B. Nonlinear observer design by observer canonical forms. INT J Control, 1988; 47(4): 1081–1100

3 Vidyasagar M. On the stabilization of nonlinear systems using state detection. IEEE Trans Automat Contr, June 1980; AC.25(3): 504–509

This paper was received for editing on November 6, 1993.
The edited paper was received on April 5, 1994.